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BEARING ESTIMATION USING RANDOM ARRAYS.(U)

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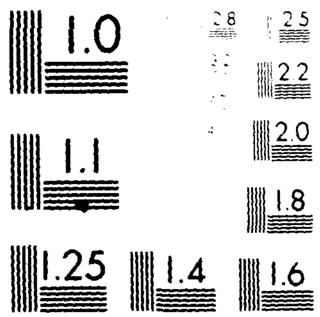

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Bearing Estimation using Random Arrays

Date: July 1982

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Abstract

A sonobuoy field is a random two dimensional array if the signals from the sensors are coherently processed. The sensor position coordinates must be estimated to a fraction of a wavelength in order to coherently process the signals, which is a separate processing task since the field has a changing random pattern due to drifting of the buoys. This paper reviews the statistical properties of the maximum likelihood bearing estimator computed from coherent summation of output signals from randomly deployed sensors. It is shown that the approximate mean-square bearing error for a small random array favorably compares with that for a square array of equally spaced sensors with about one-half the aperture. The random array is not subject to the aliasing (grating lobes) problem of an equally spaced array, and a great deal of array gain can be achieved from a large array of randomly placed sensors. Serious consideration should be given to tracking the locations of drifting sonobuoys so as to coherently process them as an array.

Bearing Estimation using Random Arrays

Melvin J. Hinich

Introduction

A sonobuoy field is a random two dimensional array if the signals from the sensors are coherently processed (Thorn, Booth, and Lockwood [1]). The sensor position coordinates must be estimated to a fraction of a wavelength in order to coherently process the signals, which is a separate processing task since the field has a changing random pattern due to drifting of the buoys. It is technically feasible to track the pattern, but the processing gain from such an effort must be perceived as sufficiently great to justify such an effort. This paper reviews the statistical properties of the maximum likelihood bearing estimator computed from coherent summation of output signals from randomly deployed sensors. Stochastic properties of the array response function are discussed by Thorn, et. al. and Steinberg [2].

The signal received at a sensor from a distant acoustic source in a horizontally homogeneous ocean is a plane wave plus ambient noise (Clay and Medwin, [3]). Using complex variable notation, a single frequency plane wave at point (x_1,x_2) and time t is

$$p(t,x) = A \exp[i2\pi f_o(t-(x_1 \cos\theta_o + x_2 \sin\theta_o)/c)]$$

where θ_o is the horizontal direction of propagation with respect to the x-axis, c is the phase velocity, f_o is the wave's frequency (the wavelength is $\lambda_o = c/f_o$) and A is the complex amplitude. This amplitude will be assumed to be a constant for the expositon of coherent

processing, but actual signals will have some amplitude and phase modulation which reduces the signal coherence across the array. A reduction of coherence results in an increase in the mean square error of bearing computed from any of the coherent processing methods discussed next.

1. Coherent Processing

The most widely used coherent processing technique is delay-and-sum beamforming. Suppose that the signal received by the kth sensor in a horizontal planar array is $s(t,\underline{x}_k) = p(t,\underline{x}_k) + \varepsilon(t,\underline{x}_k)$ where $\varepsilon(t,\underline{x}_k)$ is a Gaussian noise field at time t and point (x_{1k},x_{2k}). The usual approach for beamforming is to compute the beam output signal $B(t,\theta_\ell) = \Sigma_{k=1}^{M} s(t+\tau_{k\ell})$ for a set of delays $\tau_{k\ell} = (x_{1k}\cos\theta_\ell + x_{2k}\sin\theta_\ell)/c$ for sensors $k=1,\ldots,M$ and beam (look) angles $\ell=1,\ldots,N$. The energy in beam θ_ℓ over a sampling window of length T is $|B(\theta_\ell)|^2$ where $B(\theta_\ell)$ is the filtered beam signal in a band about f_0 of bandwidth 1/T. If the plane wave signal is broadband, then the filter passband is adjusted to the bandwidth of the signal.

Levin [4] shows that the maximum likelihood estimator of the propagation angle θ_0 is the $\hat{\theta}_0$ in the interval $0 \leq \theta < 2\pi$ that maximizes $|B(\theta)|^2$. Define

$$\hat{\sigma}_1^2 = M^{-1} \sum_{k=1}^{M} (x_{1k} - \overline{x}_1)^2, \qquad \hat{\sigma}_2^2 = M^{-1} \sum_{k=1}^{M} \qquad - \overline{x}_2)^2 \tag{1.1}$$

where $\overline{x}_1 = M^{-1}\Sigma_{k=1}^{M} x_{1k}$ and $\overline{x}_2 = M^{-1}\Sigma_{k=1}^{M} x_{2k}$. Set $\Sigma_{k=1}^{M}(x_{1k}-\overline{x})(x_{2k}-\overline{y}) = 0$

by a proper rotation of the coordinate system. Then if $\rho M\hat{\sigma}_1^2$ and $\rho M\hat{\sigma}_2^2$ are large, where ρ denotes the power signal-to-noise ratio in a narrow band about f_o, then the mean-square error of $\hat{\theta}_o$ is given by the approximation (Hinich [5])

$$E(\hat{\theta}_o - \theta_o)^2 \simeq (\lambda_o/2\pi)^2(2\rho M)^{-1}(\hat{\sigma}_1^{-2}\sin^2\theta_o + \hat{\sigma}_2^{-2}\cos^2\theta_o)\ rad^2. \qquad (1.2)$$

This approximation also holds for a discrete set of beam angles and an estimator $\hat{\theta}_N$ that maximizes $|B(\theta_\ell)|^2$, provided that the error $|\theta_{\ell+1}-\theta_\ell|$ is small for all ℓ. In many applications, however, the bias due to the discreteness of the θ_ℓ is larger than the square-root of the right hand side of (1.2). It is important in these applications to interpolate between $\hat{\theta}_N$ and its adjacent angles.

A simple way to interpolate is to use the frequency-wavenumber equivalence with beamforming (Hinich [6]). Suppose that $s(t,\underline{x}_k)$ is passed through the same narrow band filter as was used to produce $B(\theta)$. For example, the fast Fourier transform (FFT) can be used to filter discretely sampled signals to yield the filtered output

$$s(\underline{x}_k) = N^{-1}\Sigma_{n=0}^{N-1} s(n\tau,\underline{x}_k)\exp(-i2\pi mn/N), \qquad (1.3)$$

where $T = N\tau$ is the sampling period, τ is the sampling interval, and m is the integer part of f_oT. Then

$$B(\theta_\ell) = \sum_{k=1}^{M} s(\underline{x}_k)\exp[i(\kappa_{1\ell}x_{1k} + \kappa_{2\ell}x_{2k})] \qquad (1.4)$$

where $\kappa_{1\ell} = (2\pi/\lambda_o)\cos\theta_\ell$ and $\kappa_{2\ell} = (2\pi/\lambda_o)\sin\theta_\ell$. The FFT can be used to compute (1.4) for as fine a grid as desired by imbedding $\{\underline{x}_k\}$ in a square

lattice of equally spaced points, and adding zeros to expand the two dimensional array.

From now on it will be assumed that the bearing interpolation is sufficiently refined and ρM is sufficiently large so that expression (1.2) holds for $\hat{\theta}_o$. This expression will now be used to analyze the ex-ante root mean-square error of $\hat{\theta}_o$, rmse $\hat{\theta}_o$, when the sensor locations are randomly determined.

2. Gaussian Random Arrays

Suppose that the x_{1k} and x_{2k} are M independent realizations from two independent Gaussian random variables X_1 and X_2. With no loss of generality center the origin of the coordinate system so that $X_1 \sim N(0,\sigma_1^2)$ and $X_2 \sim N(0,\sigma_2^2)$, where σ_n^2 is the variance of X_n for n=1,2. To further simplify the analysis, let $\sigma_1 = \sigma_2 = \sigma$.

It then follows that $\sigma^{-2}\Sigma_{k=1}^{M}(x_{1k} - \bar{x}_1)^2$ and $\sigma^{-2}\Sigma_{k=1}^{M}(x_{2k} - \bar{x}_2)^2$ are independent chi-square χ_{M-1}^2 variates with M-1 degrees of freedom. Denote $\beta_T = \Pr(\chi_{M-1}^2 > T^2)$, e.g. $0.99 = \Pr(\chi_{15}^2 > 5.2)$ using a standard chi-square distribution table. By independence, the probability that

$$\Sigma_{k=1}^{M}(x_{1k} - \bar{x}_1)^2 > (\sigma T)^2 \quad , \quad \Sigma_{k=1}^{M}(x_{2k} - \bar{x}_2)^2 > (\sigma T)^2$$

is β_T^2, and thus

$$\Pr\{M^{-1}(\hat{\sigma}_1^{-2} \sin^2\theta + \hat{\sigma}_2^{2} \cos^2\theta) < (\sigma T)^{-2}\} > \beta_T^2 \; . \tag{2.1}$$

Applying (2.1) to (1.2) and ignoring the approximation error,

$$\text{mse } \hat{\theta}_o < (\lambda_o/8\pi\rho^{1/2}\sigma T)^2 \text{ rad}^2 \tag{2.2}$$

with probability greater than β_T^2. For example if $\beta_T = 0.99$, $M = 16$, $\sigma = 6\lambda_o$, and $\rho = 1/10$ (-10 db), then $T^2 = 5.2$ and from (2.2), rmse $\hat{\theta}_o < 0.53°$ with probability greater than 0.98.

It is useful to compare this result with the mean-square error of $\hat{\theta}_o$ for a square lattice array whose $M=N^2$ sensors are at the points $\{md,nd:m,n=1,...,N\}$. Setting $L = (N-1)d$ for the length of the square's sides,

$$\sum_{k=1}^{M} (x_{1k} - \overline{x}_1)^2 = \sum_{k=1}^{M} (x_{2k} - \overline{x}_2)^2 = ML^2/12. \tag{2.3}$$

Thus from (1.2) the mse $\hat{\theta}_o$ for the <u>random array is less than</u> that for the square lattice array (with probability greater than β_T^2) if

$$L < (12/M)^{1/2} T\sigma. \tag{2.4}$$

For example if $M=16$ and $\beta_T = 0.99$, then the bound is $L < 1.98\ \sigma$. If $M = 36$ and $\beta_T = 0.99$, $T^2 = 18.5$ and $L < 2.48\ \sigma$.

If the square is centered on the origin rather than at $((N+1)/d,(N+1)/d)$, the probability that a sensor is in the square is $[\Pr(|Y| < L/2\sigma)]^2$ where $Y \sim N(0,1)$. Thus if $L = 4\sigma$, this probability is 0.91. Since $(0.99)^2 = 0.98$, it follows from the examples above that the small Gaussian random array has approximately the same bearing accuracy as a square lattice array of about one-half the aperture.

This comparison with the lattice array has been made very conservative by the selection of $\beta_T = 0.99$. The rmse $\hat{\theta}_0$ for a small random array will often be as small as a lattice array of equal aperture.

3. Conclusion

It is shown that the approximate mean-square bearing error for a random array favorably compares with that for a square array of equally spaced sensors with about one-half the aperture. The random array is not subject to the aliasing (grating lobes) problem of an equally spaced array, and a great deal of array gain can be achieved from a large array of randomly placed sensors. Serious consideration should be given to tracking the locations of drifting sonobuoys so as to coherently process then as an array.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report 24	2. GOVT ACCESSION NO. AD-A117 531	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Bearing Estimation using Random Arrays		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Melvin J. Hinich		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-9494
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Tech Department of Economics Blacksburg, VA 24061		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 042-315
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE July 1982
		13. NUMBER OF PAGES 8
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Distribution of this document is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Random Arrays, Bearing, Beamforming, Coherent Processing		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		

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